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1993 J. Phys. A: Math. Gen. 26 L341

(http://iopscience.iop.org/0305-4470/26/6/008)

J. Phys. A: Math. Gen. 26 (1993) L341–L346. Printed in the UK

LETTER TO THE EDITOR

Dynamic scaling of disordered Ising systems

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Received 30 September 1992, in final form 4 December 1992

Abstract. The critical dynamics of site-disordered three-dimensional Ising systems has been studied by conventional local dynamics simulations investing 10^{15} single spin flips. We have verified that the correlation functions of disordered Ising systems obey dynamic scaling behaviour. The dynamic scaling functions have been calculated. The critical exponent z appears to be strongly dependent on the degree of disorder. We have shown that this dependence arises from a crossover phenomenon which has recently been detected in the static critical behaviour. The dynamical exponent has been estimated to be $z = 2.4 \pm 0.1$ which is surprisingly near to the renormalization group result in $O(\epsilon^{1/2})$.

Monte Carlo simulations have recently [1,2] revealed a complex crossover phenomenon which governs a large part of the critical region of disordered Ising systems. It has been shown that independently of the concentration the asymptotic behaviour is described by the fixed point of weak disorder [3–8]. The critical dynamics of disordered Ising systems has not been studied by Monte Carlo simulations before. Renormalization group calculations of the field-theoretic GLW-model have been performed by Grinstein *et al* [9]. They have shown that disordered spin systems obey dynamic scaling behaviour. The Harris-criterion is fulfilled in the dynamical as well as in the static behaviour. According to Grinstein *et al* [9] disorder has a large effect on the dynamical behaviour. Contrary to the pure spin system, disorder already changes the dynamical exponent $z = 2 - \eta + \Delta^*$ in first-order perturbation theory in $\epsilon = d_c - d$ around the critical dimension $d_c = 4$. As in the static case [3–8], Ising systems need a separate treatment because of the $O(\epsilon^{1/2})$-fixed point which leads to $\Delta^* = (6/53\epsilon)^{1/2}$ [9]. This extrapolates to a numerical value $z = 2.34$ in $d = 3$ dimensions which is unusually large compared with pure systems where only tiny corrections of van Hove theory ($z = 2 - \eta$) [10] have been found [11,12].

Unfortunately, the theoretical work has not been continued to higher-order perturbation theory as has been done for the static disorder problem [8]. This is a severe defect because it is well known from the statics [3–8] that higher orders in perturbation expansion are necessary to obtain reliable results in $d = 3$ dimensions. Instead, further investigations of the critical dynamics have extended the spectrum of models to include anisotropic disorder [13] and correlated defects [14–17].

In this letter we pick up the question of how the dynamics changes along the critical line $T_c(p)$ when varying the spin concentration of the site-disordered Ising

0305-4470/93/060341+06$07.50

system in three dimensions. According to the renormalization group [9] a crossover from the pure to the weakly random fixed point is expected. The asymptotic dynamics should be dominated by the fixed point of weak disorder. When the system is further diluted, the dynamics should cross over in a yet unknown way to percolative dynamical behaviour [18–23].

We have studied the dynamics by extensive Monte Carlo simulations investing about 10^{15} single spin flips in the lattice size range $L \in [20, 60]$. The concentration ranged from $p = 1.0$ to $p = 0.6$ ($p_c = 0.31$), thus covering a large portion of the critical line. Finite-size simulations have been performed at the critical temperatures $T_c(p)$ which have been determined very accurately by Binder's cumulant method [24, 2]. We have implemented the Metropolis dynamics in a vectorized form using periodic-helical boundary conditions. The dynamics may be written formally as

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}P \tag{1}$$

where $\mathcal{L}$ is the Liouville-operator acting on the probability vector $P(S)$ to find the system in some state S. The dynamical properties have been studied as usual by the estimator

$$\Phi_A(t) = \frac{\overline{\mathcal{A}(t'+t)\mathcal{A}(t')} - \overline{\mathcal{A}(t')}^2}{\overline{\mathcal{A}(t')^2} - \overline{\mathcal{A}(t')}^2} \tag{2}$$

of the normalized correlation function of thermodynamic observables [25, 26]. The bar denotes the average over the Monte Carlo time series. We have measured three quantities $\mathcal{A}(t)$: the magnetization $\mathcal{M}(t)$, its absolute value $|\mathcal{M}|(t)$ and the energy $\mathcal{E}(t)$; their different dynamical behaviour has been discussed recently [26]. In this paper we are concerned with the even quantity $|\mathcal{M}|(t)$ only, because of its high precision. The systems have been equilibrated during $20\tau_{\mathrm{int},|M|}$ Monte Carlo steps per spin. Then, the estimator (2) of the correlation function has been calculated over long runs with 1–2 $\times$ 10^6 Monte Carlo steps per spin, i.e. $O(10^4)\tau_{\mathrm{int},|M|}$. Finally, we have performed configurational averages over several hundred configurations for each concentration and lattice size. The resulting relative error which is mainly due to the configurational variance is of the order $\Delta\Phi/\Phi = 10^{-2}$ in the relevant time range where we analysed the data. Figure 1 gives an impression of the typical time dependence of correlation functions of disordered systems in comparison with the pure system including the errors.

The most remarkable feature of disordered systems is the curvature of the correlation function in the half-logarithmic plot (figure 1). While the pure system displays asymptotic one-exponential behaviour dominated by the largest relaxation time τ_{exp}, i.e. the smallest eigenvalue of the Liouville operator $\mathcal{L}$ (1), disordered systems do not reach this asymptotic limit even for long times and small values of correlations, respectively. Writing the correlation function formally as [27]

$$\Phi_A(t) = \int_0^\infty \exp(-|t|/\tau)\,\mathrm{d}\rho(\tau) \tag{3}$$

we conclude that in contrast to the pure system, there is a whole spectrum $\rho(\tau)$ of relaxation times which are relevant for the functional time dependence of $\Phi(t)$. This point will be discussed in our conclusions.

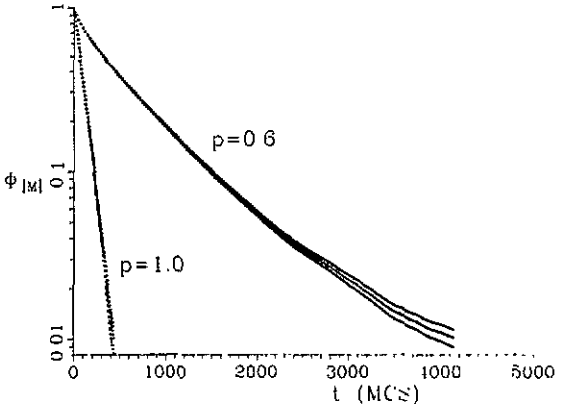

Figure 1. Half-logarithmic plot of the correlation function $\Phi_{|M|}(t)$ for the concentrations $p = 1.0$ and $p = 0.6$. Upper and lower graphs are given by $\Phi(t) \pm \Delta\Phi(t)$. Pure systems have a one-exponential relaxation below $\Phi \simeq 0.5$. Disordered systems are characterized by a strong curvature which indicates a multitude of relevant time scales.

Table 1. Numerical results of the fit of the integrated relaxation times and of the scaling method. $A_{|M|}$ is the non-universal amplitude of $\tau_{\mathrm{int},|M|}$. Both methods lead to identical values of dynamical exponents.

p	$z_{\mathrm{int},\lvert M\rvert}$	$A_{\mathrm{int},\lvert M\rvert}$	$z_{\mathrm{scal},\lvert M\rvert}$
1.0	2.095(8)	0.051	2.085(10)
0.95	2.16(1)	0.046	2.15(1)
0.90	2.232(4)	0.040	2.23(1)
0.80	2.38(1)	0.033	2.39(1)
0.60	2.93(3)	0.017	2.92(2)

Since asymptotic relaxation times could not be obtained from the data, we have calculated the integrated relaxation time [25]

$$\tau_{\mathrm{int},|M|} = \frac{1}{2}\int_{-\infty}^{\infty} \Phi_{|M|}(t)\,\mathrm{d}t\,. \tag{4}$$

It is the only quantitative measure of the relaxational processes in disordered systems. We have used the scheme of Madras and Sokal [25] (cut-off of the integral at about 6–$10\tau_{\mathrm{int}}$) as well as the scheme of Wolff [28] (exponential extrapolation of the estimator). Both methods agree within the errors. The resulting integrated relaxation times have been fitted to their finite-size scaling law $\tau_{\mathrm{int}} = A_{\mathrm{int}} L^{z_{\mathrm{int}}}$ to determine the critical exponent z_{int} and the amplitude A_{int} (table 1). We have found that the pure system has the same exponent z_{int} for integrated relaxation times as for exponential relaxation times ($z_{\mathrm{exp}} = 2.10 \pm 0.01$) [26]. The precision of the agreement is surprising. It means that short relaxation times scale with the same exponent as does the asymptotic time scale.

Our results for disordered systems (table 1) with different concentrations p show a dramatic increase of $z_{\mathrm{int}}(p)$ when disorder increases. The theoretical expectation that disorder has a very large effect on the value of z is clearly confirmed. The concentration dependence of z_{int} is very similiar to that of γ (table 1 of [2]). As we have shown previously [2], this does not violate universality but is a signature of a crossover phenomenon which affects the dynamic and the static behaviour in the same way. Thus, our values of $z_{\mathrm{int}}(p)$ (table 1) are effective exponents which reach their asymptotic value in the limit $L \to \infty$.

According to the dynamic scaling hypothesis [10], universal behaviour of the correlation function $\Phi(t, L)$ is expected for asymptotically long times. As the

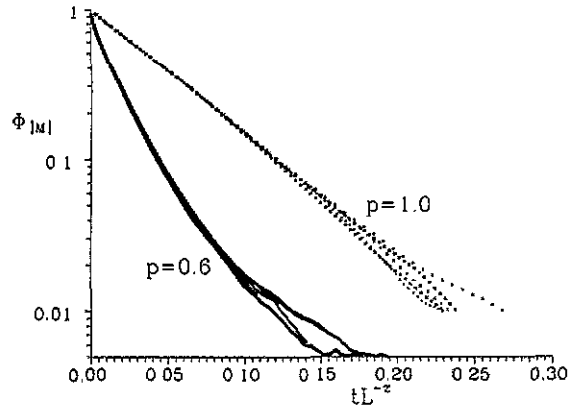

Figure 2. Scaling analysis of the correlation functions of pure and strongly disordered systems. The unrestricted data collapse taking account of statistical errors leads to precise dynamical exponents z_{scal} (table 1) and to a numerical determination of the scaling function.

integrated relaxation times τ_{int} are integrals over *all* times, it is not clear whether universal scaling is fulfilled for τ_{int} as it is for τ_{exp}. Although the coincidence of z_{exp} and z_{int} for $p = 1.0$ is quite promising, doubts may arise as to whether z_{int} actually reflects universal critical behaviour of disordered systems. These doubts are even more severe in view of the functional time dependence of $\Phi(t)$ (figure 1) which is based on a multitude of time scales.

In order to test the scaling behaviour of $\Phi(t)$ [10] given by $\Phi(t, L) = \Phi_0(tL^{-z_{scal}})$, we have done a scaling analysis of correlation functions. In our notation we distinguish the dynamical exponent z_{scal} determined by scaling from the asymptotic one. Notice that universality does not impose any restrictions on the functional dependence of the scaling function $\Phi_0(x)$ apart from the stationarity condition [29]. Since we have no information about the functional dependence of $\Phi_0(x)$ we have performed an unrestricted data collapse of our correlation functions $\Phi(t, L)$ for eight lattice sizes $L \in [20, 60]$. Details will be presented in a separate publication [30]. This scaling method leads to an unambigious determination of the exponent z_{scal}, because z_{scal} is the only free parameter to be determined by the data collapse. Based on our large number of accurate data $\Phi(t_i, L_j)$ it has been possible to apply this scaling method to intervals of Φ in order to study a possible variation of z_{int} with the scaling argument $x = tL^{-z_{scal}}$ [30].

We have obtained a collapse of all correlation functions $\Phi(t, L)$ by the appropriate value of z_{scal}. Deviations of the individual correlation functions from their scaling function $\Phi_0(x)$ is of the order of the average error (figure 2). The method is very sensitive to the correct value of z_{scal} as the errors in table 1 show. Comparison with the previously determined values of z_{int} shows perfect agreement. This is now self-understood: if $\Phi(t, L)$ scales, its integral does as well.

Summarizing, we have confirmed that disordered Ising systems obey dynamic scaling behaviour. In addition, our analysis has shown that the scaling function $\Phi_0(x)$ is highly non-trivial compared to the simple one-exponential form of the pure system (figure 2). The more disordered the system is, the stronger is the curvature of the scaling function. The scaling of $\Phi(t, L)$ implies that the integrated relaxation times themselves reflect universal scaling behaviour.

We have done a more refined analysis of the dynamical crossover which is hidden behind the concentration-dependent exponents (table 1). To this end, we have plotted the integrated relaxation times in scaling form with the anticipated

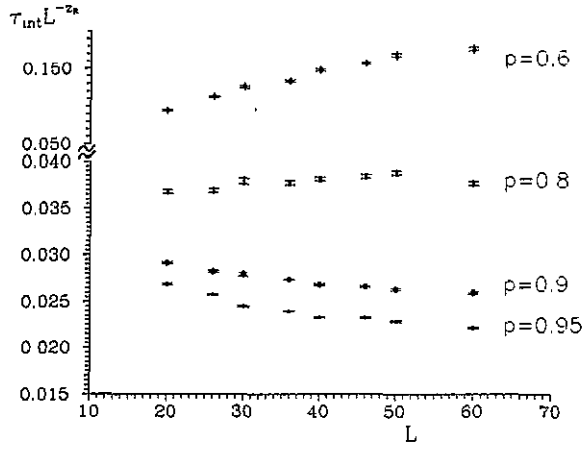

Figure 3. The scaling function $\tau_{\text{int},|M|}L^{-z_R}$ with $z = 2.34$ gives evidence of the crossover and of the asymptotic region as discussed in the text.

asymptotic exponent $z_R = 2.34$ of the weak-disorder fixed point (figure 3). In close correspondence with the static behaviour [2], figure 3 shows a crossover which governs the whole range of lattice sizes. The weakly disordered systems with $p = 0.95$ and $p = 0.9$ approach their asymptotic behaviour at lattice sizes $L \simeq 50$. The amplitude of τ_{int} in this limit is a non-universal quantity. Strongly disordered systems ($p = 0.6$) have a more extended crossover interval. Even more important is the fact that they cross over to the asymptotic behaviour from smaller amplitudes. This change of slope has already been found in the crossover of static properties. The medium concentration $p = 0.8$ which is characterized by an average exponent $z_{\text{int}} = 2.38$ (table 1) appears to be free of a crossover in that the slope of the crossover function changes its sign. Therefore, the asymptotic behaviour appears to be reached already for small lattice sizes. Our analysis strongly supports the expectation that the asymptotic behaviour is described by the weak-disorder fixed point for any concentration. Based on the close analogy of the dynamic (figure 3) and the static [2] crossover, we conclude that the true value of z_R is quite near to its first-order result [9]. We estimate that the asymptotic value is $z_R = 2.4 \pm 0.1$. A renormalization calculation to $O(\sqrt{\epsilon^2})$ would be very desirable to verify this result.

The correlation function $\Phi(t, L)$ is dominated by a multitude of time scales which contribute to its time dependence (3). Since $\Phi(t, L)$ shows scaling behaviour (figure 2), the same is true for the spectrum $\rho(\tau, L)$ (3). In contrast to the δ-like spectrum of pure systems, disordered systems have a rather broad spectrum which is depicted schematically in figure 4. A very exciting question is what is the physical origin of this unusual and non-trivial scaling behaviour. The study of random frozen configurations has shown that there are clusters of spins of varying size with strong intracoupling but weak intercoupling. These clusters have been argued to be responsible for the static behaviour in the crossover region [2]. The dynamical behaviour is similiarly influenced by these clusters which may be viewed as entities acting in a collective way. Critical spin fluctuations can be interpreted as fluctuations of whole clusters. The characteristic time for their sign reversal depends on the volume and the surface of each cluster. The correlation function $\Phi(t)$ is then given by the superposition of all clusters including their probabilities. We stress that this heuristic picture is valid only in the crossover region. If the correlation length becomes much larger than the clusters, the frozen-in structure will become irrelevant and the

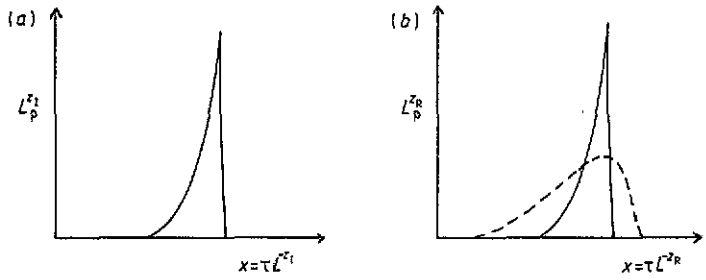

Figure 4. Schematic plot of the scaling function $L^z\rho(\tau, L) = \rho_0(x)$ with $x = \tau L^{-z}$ of the spectrum of pure (a) and disordered (b) systems. In pure systems the asymptotic relaxation time carries almost all the weight. Disordered systems have a broad spectrum (broken curve) in the crossover region where the simulation was performed. In the asymptotic region a single time scale should dominate (full curve).

scaling function Φ_0 and $\rho(\tau)$ respectively change to a simple one-exponential form. This expectation remains to be checked by simulations of larger systems.

This work was supported the Sonderforschungsbereich SFB 166 'Disorder and Large Fluctuations' and by the HLRZ.

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